4-1 Videos Guide

4-1a

- Volume using double integrals
  - $V = \iint_R f(x,y)\,dA$, where $f(x,y) \geq 0$ over R, a rectangular region in $\mathbb{R}^2$

Exercise:

Calculate the iterated integral.

- $\int_0^1 \int_0^1 (x+y)^2\,dx\,dy$

4-1b

Theorem (statement):

- Fubini's Theorem: If f is continuous or has a finite number of discontinuities on $R = \{a \leq x \leq b, c \leq y \leq d\}$, then

$$\iint_R f(x,y)\,dA = \int_c^d \int_a^b f(x,y)\,dx\,dy = \int_a^b \int_c^d f(x,y)\,dy\,dx$$

- If $f(x,y) = g(x)h(y)$, then $\int_c^d \int_a^b f(x,y)\,dA = \int_a^b g(x)\,dx \int_c^d h(y)\,dy$

Exercises:

4-1c

- Estimate the volume of the solid that lies below the surface $z = 1 + x^2 + 3y$ and above the rectangle $R = [1,2] \times [0,3]$. Use a Riemann sum with $m = n = 2$ and choose the sample points to be lower left corners.
  (b) Use the midpoint rule to estimate the volume in part (a)
- Evaluate the double integral by first identifying it as the volume of a solid.

$$\iint_R (2x+1)\,dA, \qquad R = \{(x,y) | 0 \leq x \leq 2, 0 \leq y \leq 4\}$$

4-1d

- Calculate the iterated integral.

$$\int_1^3 \int_1^5 \frac{\ln y}{xy}\,dy\,dx$$

- Calculate the double integral.

$$\iint_R (y + xy^{-2})\,dA, \quad R = \{(x,y) | 0 \leq x \leq 2, 1 \leq y \leq 2\}$$